The Fractal Lacunarity of Dry Walls and Their Matrix Stones:
Why Some Walls Look Good and Others Do Not

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Abstract

Based upon the knowledge of the observer, there may be a structural appreciation for dry stone walls. Some observers have learned, through experience, to distinguish a well built wall from one that is poorly constructed. (i.e. they know what to look for.) Apart from such a structural analysis, there is an aesthetic or artistic appreciation of any stone structure. Artistic appreciation of stone walls may involve aspects other than structural analysis. This study analyzed the effect of lacunarity (the amount and distribution of gaps between adjacent stones) on the aesthetic evaluation of stone walls. Photographs of actual walls were used for this study. The fractal lacunarity of the images of stone walls was altered using standard computerized photographic techniques. Volunteer subjects were asked to choose the "most attractive" stone wall from pairs of photographs of the same stone wall each differing from the next with altered lacunarity. Aesthetic appreciation of stone walls was found to correlate with the lacunarity and horizontal linearity within images of stone walls and the distribution of gaps/stone.

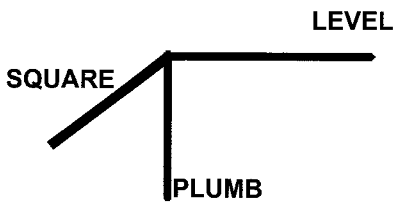
Background

The craft of building stone walls is as ancient as man. Our earliest ancestors used stones as tools and drew some stones around themselves before they slept. By coincidence these stones and their unnatural placement are all that remains of ancient man for our review. So, perhaps, the first and the lasting artifact of any human civilization are stone walls. In fact we almost define civilization by the presence of multiple dwellings and stone walls. Indeed, this dawn of history is called the Stone Age.

We have a rich cultural heritage of stone wall building. Early religions, such as the Pythagoreans, still extant today in the contemporary Masonic codes or in the Judeo-Christian legend of Joseph as a carpenter, labeled the three spatial dimensions. The adage of a good stonemason or carpenter is still **"Level, Plumb and Square."**

However, as we learn more about ourselves and the world around us, we find that these three discreet dimensions do not adequately describe everything we see. It seems that there are branching structures and iterative constructs and the shape of the gaps in stone walls that are more accurately defined using intermediate dimensional measures. These intermediate measures are called "Fractals" (Fractional Dimensions.) (5)

Three Dimensions (D = 3)



A simple example of a fractal measurement might be the length of any coast-line. The length of any finite line segment is familiar to us; it is the distance between each end-point of the line segment. The distance between two points on a real coast-line depends upon how closely we measure that distance. If we were to simply 'pace' off the distance we would count so many steps. If we were to lay a flexible measuring tape along the coast-line we would find it to be much longer than we initially counted. If we continued this example and found more precise ways to measure the coast-line, we find that the distance keeps expanding, in a manner that appears without limit, yet we know from experience that there must be some finite distance between the starting and finishing points on the coast. Thus we are faced with a line that is more than one linear dimension yet less than two planar dimensions. Its length is a fractal.

Another familiar example of a system more easily defined by a fractal dimension than a linear dimension is a branched system, such as the limbs of a tree or the shape and arrangements of river tributaries. These are not lines and they are not planes. Their description is best found somewhere between a line and a plane; Somewhere between one and two dimensions; Some fractal (or fractional) dimension.

Fractal measures abound in nature and are beginning to fill the scientific literature. Linear fractals, such as segmented lines or coastlines usually have measured fractal dimensions between 0.0 and 1.0. Planar fractals, such as the shape of a cloud's shadow, a snowflake or the branching of a silhouette of a tree exhibit fractal dimensions between 1.0 and 2.0. Three dimensional fractals and more complex lower dimensional fractals may have fractal dimensions well above 2.0. Interestingly, yet not surprisingly, people prefer certain artificial fractal relationships. Perhaps these preferences exist because such fractals appear to mimic natural fractals. Perhaps our brains are simply 'hard-wired' to apprehend and appreciate such relationships.(8) Several recent papers have been published examining the fractal dimensions of modern abstract art.(6, 7)

Observers of artistic works by Jackson Pollock seemed to favor those pieces that demonstrated a fractal dimension between 1.3 and 1.5. Others have reported that humans seem to prefer images with a fractal dimension approximating 1.6, the ancient ‘golden mean’ or ‘phi’ the limit ratio of neighboring Fibonacci numbers.(2)

Although the fractal dimension is a convenient way to characterize many images and objects, it does not, exhaustively, describe all images or objects. The properties of even a mathematically constructed fractal set are not completely described by its fractal dimension. Several images may have the same fractal dimension yet appear widely different. An example of these are plots of a group of Cantor sets each with a fractal dimension of D=1/2.

$$\mathbf{D = \log N / \log (1/r) = \log 2^{k} / \log 4^{k} = 1/2}$$

As the value of the index ***k*** varies, the distribution of the ‘space’ within these sets varies widely. The notion of this space (the gaps between numbers or substances) is called lacunarity. In this example we see that lacunarity may vary widely within the same fractal dimension. Several measures of lacunarity have been proposed and a common definition based upon light and dark regions within a given window of observation is:

$$\lambda (r) = \text{variance} / (\text{mean})^{2}$$

Where lambda is the lacunarity and r is the window size. In a relatively smooth or homogeneous fractal set, where $\lambda \rightarrow \mathbf{1}$, gap size and variation is minimal. As $\lambda \rightarrow \infty$ a wide distribution and range of gap sizes appears creating a heterogeneous texture.(1, 4)

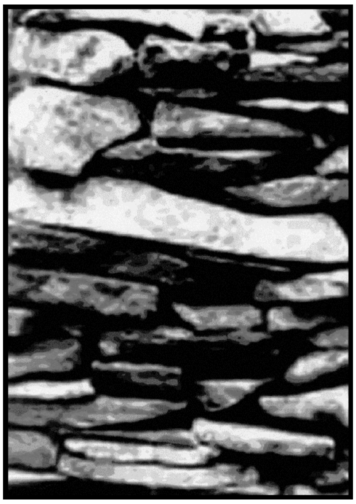
Stone walls are relatively easy to define yet surprisingly complex structures. They are piles of stones placed by humans. This very broad definition includes piles of rubble that may mimic natural accumulations of stones. To the eye stone walls are stones and gaps between these stones; Areas of light and shadow.

Apart from the utility of stone walls and the use of stone as a building material, we also notice a certain aesthetic appreciation of stone structures.(3)

Study

The question addressed in small part by this paper is why some stone walls look good and others do not and how is this related to the distribution of space, light and shadow within a stone wall. By examining the lacunarity of stone walls and correlating this to a stated human preference, we show that there is an aesthetic appreciation of stone walls linked to the distribution of stones within those walls.

Methods



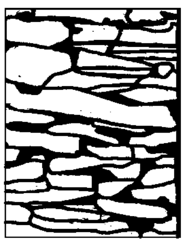
Subjects were presented with photographic images of actual stone walls. Some of these images were altered, using standard computerized photographic techniques to effect the light and shadow contrast and the orientation of the image. Sections or windows of these images were chosen for presentation to eliminate any consideration of the overall design of the original wall structure. (We did not want to have to unravel any preference for aquaducts over stockades.) Images were presented to subjects as pairs. The fractal dimension of each pair was held close to constant by these photographic transformations. The pairs of images were presented so that each image of any pair varied from its neighbor by only one variable (i.e. contrast ratio, orientation or lacunarity.) Essentially, the lacunarity of each image was adjusted while care was taken to maintain the fractal dimension of the original image.

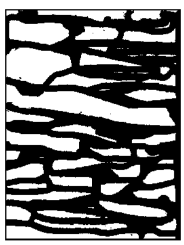
Ten original images of historic and contemporary dry stone walls were presented as eight images in pairs for each original image. The four pairs of images represented, first, a slightly altered contrast between light and dark areas within the image, second, a 90^{o} rotation of the image, third, a complete reversal of contrast (a negative image,) and, fourth, the image with thicker shadow lines.

1E

1F

1G





1H

The subjects were asked to choose one image of each pair that they found “more attractive” or “better looking” than the other.

Standard statistical methods were applied to these data reports.

Results

The demographics of the subjects participating in this study were:

Table 1

	Males	**Females**	**Total**
Number	32	36	68
Age Range	15-56	16-77	15-77
Mean Age	29	29	29

First we looked at the variables chosen to present lacunarity. There was no significant difference in the selection between two 20/50 and 50/20 contrast/brightness images. There was no significant difference in the selection between any positive and negative images (those with white and black areas transposed.) There was a significant difference in the selection of thin-lined images over thick-lined images ($p<0.01$). There was also a significant difference in the selection of rotated images ($p<0.001$) with the original horizontal image being preferred over the rotated vertical image.

A regression analysis was performed on the data representing the selection of horizontal walls over vertical walls (Orientation), assuming there was a learned preference for horizontally placed stones over vertically placed stones. However, while the data show that the older subjects chose the horizontal over the vertical more often than the younger subjects (with a very low $\mathbf{r}^2 = 0.0055$) any learned effect would take almost 50 years to assure such a selection. This was not statistically significant. Some subjects simply prefer horizontal lines over vertical lines. (Perhaps this is why some of us are stone masons and others are not.)

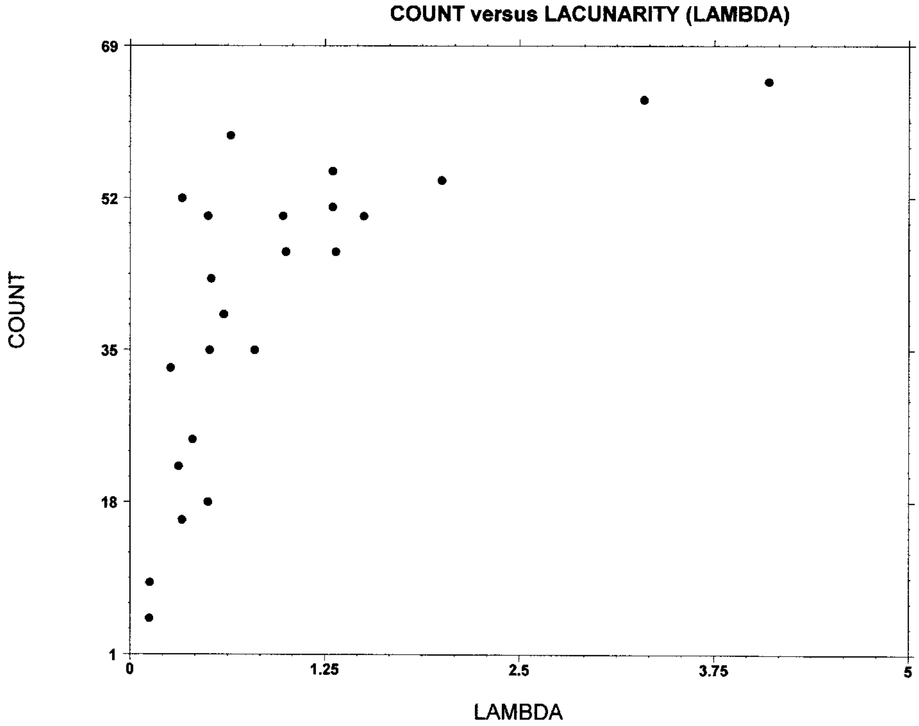
The lacunarity of several of the sample images was measured. Sample images were selected from representative portions of the response scale (high, low and mid-range counts.) Lacunarities for these images ranged from 0.1 to 4.0. The measured lacunarity was plotted against the number of counts for each image. We found, very clearly, that those images representing the more frequently selected images had higher lacunarities than those images selectively ignored by the subjects. A non-linear regression analysis was not performed on these partial data. However, it is evident that subjects selected images with higher lacunarities far more often than those with lower lacunarities. This seems to be an open ended relationship. Subjects preferred an image with more texture rather than smooth images.

Conclusions

Aside from function there is an aesthetic appreciation of stone walls. This appreciation or preferential selection of some images of stone walls over similar images correlates with horizontal lines and the lacunarity of the image. A higher lacunarity or diversity of texture coincides with a favorable impression. For actual dry stone walls this finding implies that given two similar stone walls the one that exhibits the inclusion of some very large stones and more horizontal lines will appear more attractive than the one that has a uniform texture and more vertical lines.

Table 2

	N=68 Subjects	***Count***	***Mean***	***p***
	(10 Counts/subject)		(per subject	
Contrast	20/50	447		
	50/20	233	ns	0,06
Contrast	positive	330		
	negative	350	ns	0,69
Orientation	horizontal	491	7,2	
	vertical	189	2,8	<0.001
Lacunarity	thin line	386	5,7	
	thick line	294	5,3	0,01



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